A stabilized finite element model for the hydrothermodynamical simulation of the Rio de Janeiro coastal ocean

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SUMMARY

In this paper, a stabilized space–time Petrov–Galerkin finite element model of coastal ocean circulation is derived. Continuous linear interpolation in space and time is employed and a spatial unstructured mesh of triangular elements is used. To simulate the hydro-thermodynamical ocean behaviour a gravity reduced model is proposed. This model solves the motion and continuity hydrodynamic equations coupled with the advection–diffusion transport equation for temperature which governs the heat exchanges in the upper layer. The model is used to evaluate the main dynamical features of the coastal waters of the Rio de Janeiro State, namely: cold plumes, coastal jets and the influence of the warmer water bays. Numerical experiments are performed to highlight the typical observed events of coastal upwelling phenomena and their interaction with the existing coastal bays and the offshore conditions. Such experiments deal with a series of idealized northeast wind field forcing patterns, showing the solutions dependence on the horizontal structure of the wind fields in relation to the coastline configuration. Copyright © 2007 John Wiley & Sons, Ltd.

1. INTRODUCTION

Finite element approximations of oceanography problems require the use of well non-uniform refined mesh to take into account the irregular coastline shape. One of the main difficulties using finite element models is the unphysical wave scattering due to changes in grid spacing,
combined with convection-dominated phenomena. Related numerical spurious oscillations could be effectively suppressed or highly attenuated, using a consistent variational Petrov–Galerkin (PG) formulation to improve the classical finite element Galerkin approximation. This approach gives rise to the well-known ‘streamline upwind Petrov–Galerkin’ (SUPG) method, and other related shock-capturing operators for advection–diffusion and compressible flow problems, (e.g. [1–4]). Other PG formulations using a symmetric form of the shallow water wave equations were reported in the literature, in terms of the so-called entropy variables [5, 6] or in terms of velocity/celegy variables [7, 8]. For hydrodynamic baroclinic circulation a similar approach was recently presented in [9], focusing the wind-driven ocean dynamic response in open and limited areas.

The coastal region of Rio de Janeiro State (Figure 1) is located in the Central-Southern Brazilian littoral. Along this coastline, the Guanabara Bay, Sepetiba Bay and Ilha Grande Bay are the most important hydrological features.

The Brazil Current (BC), with sea surface temperatures (SST) that may range from 25 to 27°C during the summer and from 22 to 24°C during the winter, occupies the mid and outer shelf waters of the indicated region. The BC is a warm, upper-layer current flowing southward, adjacent to the eastern continental shelf of South America, which originates at the divergence of the South Equatorial Current in the equatorial region.

The influence of the BC, along the entire southern coast, is stronger in summer than in other seasons (e.g. [10]). The salinities normally range from 36.5 to 37.0% [11]. The source of the upwelling is the South Atlantic Central Waters (SACW). Typically, the conditions of SACW are recognized by low temperatures (<18°C) and high levels of nitrate concentrations from 4 to 10 μM (1μM = 1 μg atoms of nitrogen/litre) [12].

The presence of SST anomalies, caused by wind-driven coastal upwelling, is a typical pattern around the Cabo Frio Cape (to the east of Rio de Janeiro coast), particularly during the austral summer. Another common dynamical feature in this region is the sequence of upwelling–downwelling
events. The cooling pattern is characterized by cold plumes of SST, which extend up to 30 miles offshore [13]. When northeast prevailing winds persist for several days, divergent flows and SST drops to around 15°C are produced during upwelling phases. The SST increases when the wind rotates and flows from southwest due to the passage of frontal systems. Converging flows and warm water intrusions take place during the downwelling phases [13]. The upwelling–downwelling events are in contrast to the dynamics found along typical upwelling regions (e.g. Perú, Oregon).

From the analysis of remote sensing images (AVHRR SST), it was reported by Carvalho [14] that during summer the upwelling plumes have the tendency to be mainly concentrated between the cape of Cabo Frio and the Guanabara Bay (Plate 1(a)). The plumes are accompanied by the formation of frequently elongated filaments in southwest direction. During winter the upwelling plumes are larger, covering the coastal band from the Cabo Frio Cape to Ilha Grande Bay (Plate 1(b)). Filaments are still generated, but they are not so elongated as those observed in summer.

The surface circulation could be inferred from the SST images reproduced in Plates 1(a) and (b). From these figures we observe the presence of cold-water plumes in offshore direction (oriented to southwest) which means that in these sectors the wind-driven offshore velocity components must be stronger than in the surrounding area, pointing out the importance of non-uniform spatial wind patterns. Near the coast of Rio de Janeiro, the upwelling front propagates quite faster reaching a distance of 30 miles offshore [13] in around 2 days. This means that, in these areas the advection velocities must be around 0.2 m s$^{-1}$ or greater.

Numerical studies concerning the SST fields and the associated circulation patterns have shown that the characteristic cold plumes of SST, which extend up to 30 miles offshore, are dynamically forced by non-uniform wind distributions [15]. In the same manner, the typical observed sequence of upwelling–downwelling events in the Cabo Frio region is also related with the change on the dominant direction of the wind fields [16]. The wind-driven hydrodynamic strongly contributes to the occurrence of phytoplankton blooms in this region [17].

In summary, the coastal region of Rio de Janeiro exhibits a complex interaction between major currents (BC), the wind-driven coastal upwelling of Cabo Frio and the coastal bays. Such interaction gives rise to a variety of physical processes near the coast, including the generation of cold upwelling plumes, warm plumes and strong fronts. The hydro-thermodynamics heterogeneity near the coast may result from the hydrodynamic and thermodynamic forcings, whose dynamical consequences on frequently changing patterns have not yet been deeply examined or numerically simulated for this region.

Aiming to a better understanding of those hydro-thermodynamical interactions, in the present paper we focus our numerical study on the behaviour of the SST and current fields, in the nearshore area of the Rio de Janeiro coast, using a PG coastal model forced by typical schematized non-uniform wind fields.

### 2. THE MODEL

#### 2.1. The governing equations

We use Cartesian co-ordinates $(x, y) \in \mathbb{R}^2$ oriented positively to the east and north, respectively, and focus on a limited coastal ocean region defined in a domain $\Omega \subset \mathbb{R}^2$. We consider land-type boundary $\Gamma_L$ and ocean-type boundary $\Gamma_O$, such that $\Gamma_O \cup \Gamma_L = \Gamma$ and $\Gamma_O \cap \Gamma_L = \emptyset$. To describe the vertical structure, two layers are considered: the upper surface layer of density $\rho_u$, where
hydrodynamic features are considered; and the inert lower layer $\rho^l$, where it is assumed that the horizontal pressure gradient is zero. As a consequence, the faster barotropic mode is eliminated and just the first internal baroclinic mode is considered. These assumptions give rise to the so-called gravity reduced model (e.g. [18, 19]).

The thermal structure is defined by the instantaneous upper-layer temperature $T$, and the lower-layer temperature $T^l$. As a constitutive state equation we assume the ansatz $\rho^u = \rho^l [1 - \alpha (T - T^l)]$, where $\alpha$ is the thermal expansion coefficient. This equation does not consider the salinity effects, but there is no loss of generality, since an apparent temperature could be estimated modifying the coefficient $\alpha$, to include the salinity concentration effect [20, 21].

From the previous considerations the vertically integrated hydro-thermodynamic equations for the coastal ocean can be written as

\begin{align}
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + g \left( \frac{\partial h}{\partial x} + \frac{h x \partial T}{2\mu} \right) - f v - \vartheta \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) - \tau_x \rho^l h = 0 \tag{1a} \\
\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + g \left( \frac{\partial h}{\partial y} + \frac{h x \partial T}{2\mu} \right) + f u - \vartheta \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) - \tau_y \rho^l h = 0 \tag{1b} \\
\frac{\partial h}{\partial t} + u \frac{\partial h}{\partial x} + v \frac{\partial h}{\partial y} + h \frac{\partial u}{\partial x} + h \frac{\partial v}{\partial y} - w_{ed} = 0 \tag{1c} \\
\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} - \vartheta_T \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + \frac{1}{h} (Q_I - Q) = 0 \tag{1d}
\end{align}

In the above expressions $u$ and $v$ are the $(x, y)$-velocity components in the upper-layer; $h$ is the upper-layer thickness; $f$ is the Coriolis parameter that depends on the geographical co-ordinates; $g$ is the gravity acceleration; $\vartheta$ represents the eddy viscosity; and $\vartheta_T$ is the thermal diffusivity. The entrainment–detrainment velocity $w_{ed}$ is assumed to be: $w_{ed} = (H_e - h)(H_e - h)/t_e$, where $H_e$ and $t_e$ represent the depth and the time scales of the entrainment–detrainment process (they are defined here in a similar way as done by McCreary and Kundu [18]). The wind stress components are represented by the quadratic relations $\tau_x = c_w \rho_{air} Wx |W|$, $\tau_y = c_w \rho_{air} Wy |W|$, where $Wx$ and $Wy$ are the corresponding wind velocity components, and $|W|$ represents the wind velocity intensity (the drag coefficient $c_w$ is evaluated following Enriquez and Friehe [22]). Finally, the parameters $\sigma$ and $\overline{\mu}$ are defined as: $\overline{\mu} = \mu / (\mu - \sigma)$; $\sigma = 1 - \mu$; $\mu = \rho^u / \rho^l$.

According to Equation (1d) the thermodynamic behaviour of the model is driven by the surface heat flux $Q$ between the ocean and the atmosphere, and by the interface heat flux $Q_I$ between the two ocean layers considered. The surface heat flux is expressed by: $Q = (H^2 / t_T) (T^u - T) / h$, where $H$ is the initial upper-layer thickness and the parameter $t_T$ represents the time for the upper layer to relax back to the initial temperature $T^u$. The surface heat flux definition adopted for $Q$ follows the parameterization proposed by McCreary and Kundu [18], which is similar to the one proposed by Haney [23].

The gain or loss of heat across the internal ocean interface, which depends on the dynamic convergence or divergence of the flows $\omega$ in the upper layer, is given by $Q_I = k_I (T - T^l) / h$, where the parameter $k_I = \omega H$ and $\omega = - (\partial h / \partial t - w_{ed})$ [16].
A more convenient form for the governing Equations (1) can be obtained if we introduce a new set of ‘celerity’ variables, namely

\[ d = 2c, \quad c = \sqrt{g\sigma h}, \quad b = \frac{c^0 T}{Tu} \]  

(2)

where \( c^0 = \frac{\sqrt{g\sigma H}}{2} \) is a referential celerity for the initial upper-layer thickness \( H \).

Introducing a column array vector of unknown variables \( V = (u, v, d, b)^T \), and a new parameter \( s = ghxTu/2\pi c^0 \), the previous system of Equations (1) can be rewritten as

\[
\frac{I V}{pt} + \frac{A}{x} V + \frac{B}{y} V + C V + D \left( \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} \right) + F = 0
\]

(3)

\[
A = \begin{pmatrix}
u & 0 & c & s \\
0 & u & 0 & 0 \\
c & 0 & u & 0 \\
0 & 0 & 0 & u \\
\end{pmatrix}, \quad B = \begin{pmatrix}v & 0 & 0 & 0 \\
0 & v & c & s \\
0 & c & v & 0 \\
0 & 0 & 0 & v \\
\end{pmatrix}
\]

\[
C = \begin{pmatrix}0 & -f & 0 & 0 \\
f & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{pmatrix}, \quad D = \begin{pmatrix}0 & 0 & 0 & 0 \\
0 & -\vartheta & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & -\vartheta_T \\
\end{pmatrix}
\]

\[
F = \begin{pmatrix}-\tau_x/\rho^u h \\
-\tau_y/\rho^u h \\
-weg/c \\
(Q_1 - Q)/h \\
\end{pmatrix}, \quad V = \begin{pmatrix}u \\
v \\
d \\
b \\
\end{pmatrix}, \quad I = \begin{pmatrix}1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
\end{pmatrix}
\]

(4)

2.1.1. Boundary and initial conditions. For land-type boundaries, we prescribe the non-slip conditions

\[ u = v = 0 \]

(5)

and Neumann boundary conditions are assumed for both the upper thickness \( h \) and temperature \( T \).

For the open ocean boundary we prescribe a weakly reflective boundary condition based on the characteristic method. This is done assuming that along the characteristics \( dx_n/dt = u_n \pm c \), the Riemann invariant \( R^\pm = u_n \pm d \) satisfies

\[
\frac{dR^\pm}{dt} = 0
\]

(6)
where \( u_n \) represents the \((x_n)\)-velocity component in the normal direction to the open boundary. The weakly reflective conditions on the open boundary are then defined by the in-going characteristic of the presented equations. (See [16] where the advantages of this kind of open boundaries modelling coastal upwelling were recently reported.)

Finally an appropriate initial state should be assumed, such that

\[
\begin{align*}
    u & = u^0, \\
    v & = v^0, \\
    d & = d^0, \\
    b & = b^0 \quad \text{in } \Omega
\end{align*}
\]  

(7)

where \( u^0, v^0, d^0 \) and \( b^0 \) represent the initial values at \( t = 0 \). The variables \( d^0 \) and \( b^0 \) are obtained from the initial thickness \( H \) and temperature \( T^u \) according to Equation (2).

2.2. Space–time Petrov–Galerkin method (STPG)

To approximate the ocean problem previously presented we define a space–time finite element partition \( \pi^{h,\Delta t} \); such that the time domain interval \([0, \Sigma]\) is partitioned into subintervals

\[
I_n = t_{n+1} - t_n = \Delta t, \quad t \in [0, \Sigma]
\]

(8)

where \( t_n, t_{n+1} \) belong to an ordered partition of time levels \( 0 = t_0 < \cdots < t_n < t_{n+1} < \cdots < t_F = \Sigma \).

For each \( n \) the space domain \( \Omega \) is partitioned in \( N \) sub-domains \( \Omega_e \) with boundary \( \Gamma_e \), such that

\[
\Omega_i \cap \Omega_j = \emptyset \quad \text{for } i \neq j, \quad i, j = 1, \ldots, N
\]

(9a)

\[
\bigcup_{e = 1}^{N} \Omega_e = \Omega
\]

(9b)

As a result, for each \( n = 1, 2 \ldots \) the space–time domain of interest is the slab \( S_n = \Omega \times I_n \) with boundary \( \Gamma_n = \Gamma \times I_n \); the lateral surface of this ‘slab’.

Under the above definitions, we will assume that the finite element subspace of weighting functions is the set of continuous piecewise polynomials \( \hat{V}^h \) in \( S_n \), i.e.

\[
\hat{V}^h_n = \{ \hat{V}^h; \hat{V}^h \in (C^0(S_n))^d, \hat{V}^h|_{\Omega_e} \in (P^k(\Omega_e))^d, \hat{V}^h|_{\Gamma_n} = 0 \}
\]

(10)

Note that the previous definition implies that in the spatial domain each component of the vector \( \hat{V}^h \) has \( C^0 \) continuity; its restriction to a particular finite element \( \Omega_e \) being a polynomial of degree less than or equal to \( k \). Across the slab interfaces those components may be discontinuous. Nevertheless, in this work it will be assumed that the variables change continuously in time, and a continuous linear interpolation in space and time will then be adopted.

Now let us introduce a vector \( \hat{V}^h \) of prescribed boundary conditions on \( \Gamma_n \). As a consequence, the set \( U^h_n \) of admissible trial functions will be

\[
U^h_n = \{ V^h; V^h \in (C^0(S_n))^d; V^h|_{\Omega_e} \in (P^k(\Omega_e))^d; V^h|_{\Gamma_n} = \hat{V}^h \}
\]

(11)

According to the previous definitions, the space-time Petrov–Galerkin (STPG) approximate solution for the hydro-thermodynamic problem is the vector \( V^h \in U^h_n \), such that

\[
\int_{S_n} \hat{V}^h \cdot R^h \, d\Omega \, dt + \sum_{e = 1}^{N} \int_{S_e} \Psi \tilde{L}_S \hat{V}^h \cdot R^h \, d\Omega \, dt = 0 \quad \forall \hat{V}^h \in \hat{U}^h_n
\]

(12)
where

\[ R^h = \tilde{L}V^h + CV^h + D \left( \frac{\partial^2 V^h}{\partial x^2} + \frac{\partial^2 V^h}{\partial y^2} \right) + F \]  

(13)

is the residual vector; \( \Psi \) is the matrix of intrinsic time scales

\[ \tilde{L} = I - \frac{\partial}{\partial t} + A \frac{\partial}{\partial x} + B \frac{\partial}{\partial y}, \quad \tilde{L}_S = I - \frac{\partial}{\partial t} + A_S \frac{\partial}{\partial x} + B_S \frac{\partial}{\partial y} \]  

(14)

and \( A_S, B_S \) are the symmetrical counterparts of the original non-symmetric matrices \( A \) and \( B \), with \( s = 0 \) (Equation (4)). These matrices are functions of \( V^h \).

The elements of the symmetric and positive semi-definite matrix \( \Psi \) are free parameters, commonly known as upwind functions. They could be interpreted as ‘scaling parameters’ which should be adequately chosen to ‘adjust’ the weighting function effect. An optimal choice for \( \Psi \) is still an open problem. Convergence analysis and error estimates can provide some design conditions for \( \Psi \), but are insufficient to determine an unique definition. For the numerical experiments that will be presented later, the matrix \( \Psi \) was chosen as

\[ \Psi = \tau I \]  

(15)

where \( \tau \) represents an intrinsic time-scale parameter. Numerical experiments presented in [8] indicate that the effect of increasing the free parameter \( \tau \) in the corresponding STPG formulation is to reduce the excessive damping introduced by the space–time continuous Galerkin formulation (an over-damped scheme provided by the first integral term in Equation (12)). In the present study, the free parameter \( \tau \) is constant.

Let us now be restricted to a generic space–time finite element \( S^n_e = \Omega_e \times \Delta t \) (a tetrahedral element), with nodal variable values \( V^e \) and local space–time interpolation function \( \Theta \). Then, the approximate solution in this element is written as

\[ V^h_e(x, y, t) = \Theta^T V^e \]  

(16)

where

\[ V_e = (V^{n+1}_e, V^n_e)^T = (V^{n+1}_1, V^{n+1}_2, V^{n+1}_3, V^n_1, V^n_2, V^n_3)^T \]  

(17)

\[ \Theta^T = (\Theta^{n+1}, \Theta^n)^T = (\Phi \theta, \Phi(1 - \theta))^T \quad \Phi^T = (\Phi_1, \Phi_2, \Phi_3)^T \]  

(18)

and \( \Phi_i \) \( (i = 1, 2, 3) \) are the spatial linear polynomials for triangular finite elements, and \( \theta \) and \( (1 - \theta) \) are the linear time interpolation functions associated with levels \( (n + 1) \) and \( n \), respectively.

Substitution of (17) and (18) into the variational form (12) leads to a system of algebraic equations of the form

\[ MV^{n+1} = -(NV^n + F) \]  

(19)

where \( V^{n+1} \) is the global vector of unknown variables at time level \( n + 1 \) and \( V^n \) the global vector of known variables at time level \( n \).
The global matrices $M$ and $N$ assembled from the element matrices as well as the vector $F$ which contains known quantities associated with the source terms are represented as

\[
M = \sum_{i=1}^{N} \left[ \int_{\Omega_{e} \times \Delta t} (\Theta^{n+1} \tilde{L}^{e} (\Theta^{n+1})^T + \Theta^{n+1} \tilde{C}^{e} (\Theta^{n+1})^T) \, d\Omega \, dt \right]
\]

\[
- \int_{\Omega_{e} \times \Delta t} \left( \frac{\partial \Theta^{n+1}}{\partial x} D^{e} \left( \frac{\partial \Theta^{n+1}}{\partial x} \right)^T + \frac{\partial \Theta^{n+1}}{\partial y} D^{e} \left( \frac{\partial \Theta^{n+1}}{\partial y} \right)^T \right) \, d\Omega \, dt
\]

\[
+ \Psi^{e} \int_{\Omega_{e} \times \Delta t} \tilde{L}^{e} \tilde{S} \Theta^{n+1}.(\tilde{L}^{e} (\Theta^{n+1})^T + \tilde{C}^{e} (\Theta^{n+1})^T) \, d\Omega \, dt \]  

(20)

\[
N = \sum_{i=1}^{N} \left[ \int_{\Omega_{e} \times \Delta t} (\Theta^{n+1} \tilde{L}^{e} (\Theta^{n})^T + \Theta^{n+1} \tilde{C}^{e} (\Theta^{n})^T) \, d\Omega \, dt \right]
\]

\[
- \int_{\Omega \times \Delta t} \left( \frac{\partial \Theta^{n+1}}{\partial x} D^{e} \left( \frac{\partial \Theta^{n+1}}{\partial x} \right)^T + \frac{\partial \Theta^{n+1}}{\partial y} D^{e} \left( \frac{\partial \Theta^{n+1}}{\partial y} \right)^T \right) \, d\Omega \, dt
\]

\[
+ \Psi^{e} \int_{\Omega_{e} \times \Delta t} \tilde{L}^{e} \tilde{S} \Theta^{n+1}.(\tilde{L}^{e} (\Theta^{n})^T + \tilde{C}^{e} (\Theta^{n})^T) \, d\Omega \, dt \]  

(21)

\[
F = \sum_{i=1}^{N} \int_{\Omega_{e} \times \Delta t} (\tilde{L}^{e} (\Theta^{n+1})^T) \, d\Omega \, dt
\]

(22)

where $\tilde{L}^{e}$, $\tilde{L}^{s}$, $\tilde{C}^{e}$, $D^{e}$, $\Psi^{e}$, respectively, denote the extensions of matrices $\tilde{L}$, $\tilde{L}^{s}$, $C$, $D$, $\Psi$ for a generic element ‘$e$’.

Remark
This model can describe strong variable gradients, guaranteed the wavelength be much larger than the upper-layer thickness. On the other hand, drastical changes of the upper-layer thickness are out of the model scope since in this situation important vertical motions should occur, and this is in disagreement with the basic shallow water wave assumption of hydrodynamic motion perturbation of a vertical hydrostatic equilibrium configuration. Therefore, for the numerical experiments that will be presented in the next section, the adopted linearization procedure approximates the matrix $M$ in terms of the known variables $V^{n}$, which is acceptable for small time-steps and long wave solutions.

3. MODELLING EXPERIMENTS

All numerical experiments reported in the present section were performed in an ocean region which extends from the 22 to 25\(^{\circ}\)S in the north–south direction, and from the 41 to 45\(^{\circ}\)W in the east–west direction. The parameter values used in these experiments are listed below.
The densities of the upper and lower layer are, respectively, $\rho^u = 1024 \text{ kg m}^{-3}$ and $\rho^l = 1025.5 \text{ kg m}^{-3}$, and the air density is $\rho_{\text{air}} = 1.175 \text{ kg m}^{-3}$. The initial upper-layer thickness is $H = 30 \text{ m}$. Observations show that during spring and summer months the interface in this area is at a depth of 20–30 m (e.g. \cite{10, 12}). The entrainment depth is $H_e = 30 \text{ m}$ and the entrainment time-scale $t_e = 0.5 \text{ days}$. The initial upper-layer temperature $T^u = 26^\circ \text{C}$, and the lower-layer temperature $T^l = 14^\circ \text{C}$, are representative values for the summer months \cite{10}. The values of the eddy viscosity and temperature diffusivity are $\vartheta = 200 \text{ m}^2 \text{s}^{-1}$ and $\vartheta_T = 50 \text{ m}^2 \text{s}^{-1}$, respectively. The entrance of each bay (Guanabara Bay, Sepetiba Bay and Ilha Grande Bay) is considered an open hydrodynamic boundary with a constant temperature of 26$^\circ \text{C}$.

The unstructured finite element mesh of the study region is depicted in Figure 2. The discretized domain corresponds to 2705 nodes and 5146 triangular elements. It can be observed that near the coastline a very refined mesh is used, which becomes coarser near the offshore region. The smaller sides of the elements along the coastline are around $\sim 1852 \text{ m}$ (1 nautical mile), whereas in the outer boundary the element sides are up to 5556 m (3 nautical miles).

The calculation time-step is $\Delta t = 1200 \text{ s}$. Note that for the adopted mesh a Courant number $Cr = 1$ leads to a time-step range of 1000–3000 s. The intrinsic time-scale parameter is prescribed as $\tau = 600 \text{ s}$, which for the numerical experiments under consideration is an acceptable value to describe both smoothed and sharp waves \cite{8}. For smoothed waves the values of $\tau$ could be greater; but for sharp waves acceptable values of $\tau$ should be less than the time-step corresponding to $Cr = 1$.

All numerical experiments were started from rest with no horizontal pressure gradients at $t = 0$ and integrated forward in time during 5 days. The model is driven by steady northeast wind stress fields as shown in Figure 3. The four schematic spatial configurations, adopted in this work for the winds, were inspired on some common observed wind patterns provided by the QUICKSCAT satellite (Julian days: 9, 301 at night pass, and 19, 15 at day pass) \cite{24}. They were used here as the first attempt to numerically explore the influences of such wind field patterns. The results of the SST and current fields at day 5 are presented in the next sub-sections.
The numerical experiments were performed in a Microcomputer AMD Athlon, of 2.21 GHz with 512 MB of RAM. The Gauss elimination model solver spent 5684 s to reach the solution at \( t = 5 \) days (360 time steps).

### 3.1. SST spatial changes

The numerical results obtained for the typical northeast wind fields considered here are in accordance with the observed behaviour. After 5 days of integration the SST decreases along the coastline, as it was expected for a northeast wind stress field, as a consequence of offshore Ekman transport in the upper layer. The common interpretation of coastal upwelling is that it is due to a wind-induced divergence mass transport of the surface water away from the coast (Ekman transport), leading to a vertical motion bringing colder and nutrient-rich water to the surface.

Figure 4 shows the SST patterns at day 5 for each wind spatial case presented in Figure 3. Cores of cold water dominate the SST field solutions and an appreciable alongshore variation is noted. Positioning of the intense upwelling cells depends on the horizontal structure of the winds relative to the coastline configuration.

When the centre of higher wind magnitudes of a wind patch are applied in the east offshore side, superposed on a moderate background wind field (Figure 3(a)), the solution shows a
coastal upwelling core around the Cabo Frio cape, which extends in the wind direction offshore (Figure 4(a)).

When the centre of the wind patch is located on the coast (Figure 3(b)), in the neighbourhood of Cabo Frio cape, the solution shows the generation of a cold plume oriented to the southwest, partially covering the coastal marine band to the west (Figure 4(b)). Along the zonal section, the temperatures increase to the west.

On the other hand, when the centre of the wind patch is located offshore (Figure 3(c)), the SST solution shows an upwelling centre in the neighbourhood of Cabo Frio cape coast and a cold-water filament extended in southwest direction (Figure 4(c)). Warmer temperatures of order of 26°C dominate the east side and the west side of this filament.

For the low wind forcing patch, presented in Figure 3(d), the numerical solution depicted in Figure 4(d) shows a perceptible drift of coastal colder water (weak Cabo Frio upwelling in origin) in offshore direction, forming cold cells of coastal advected waters in the offshore side.

The offshore conditions of warmer temperatures dominate the SST patterns in the southeast side excepting in the low wind patch case (Figure 4(d)). Also, the presence of warmer waters (offshore condition and water bays in origin) is clearly detected in the west side of the study region. For the first three cases, the border limit of influence of the offshore conditions is defined by the wind patch location. To the east of the northeast wind patch, the hydro-thermodynamic model develops a temperature front.

It is interesting to comment that the SST fields images presented in Plate 1(a) (at 18th January 2001) and Plate 1(b) (29th October 2001) had wind patterns (not presented here) similar to those shown in Figures 3(b) and (c).

3.2. Current fields

The spatial variation of wind stress produces a correspondent variation in the resulting Ekman transport in the ocean surface layer, and an associated change on the regions of hydrodynamic convergence and hydrodynamic divergence are observed.

Figure 5 shows the upper-layer velocity fields obtained after 5 days of integration, for the same four forcing wind fields previously presented in Figure 3. As expected from the coastal jet dynamics, the upper-layer currents show coastal jets along the coastline, in addition to the strong current velocities in the sector where the wind patches were intense.

The velocity magnitudes depend on the location of the centre of high wind velocities relative to the coastline. The maximum velocities were 0.37, 0.68, 0.31, and 0.52 ms$^{-1}$ for the four studied cases (Figures 5(a)–(d), respectively). The higher velocities observed in Figure 5(b) could be a consequence of the fact that the centre of the wind patch velocities of 12 ms$^{-1}$ be located on the

Plate 1. Observed SST field by AVHRR sensor for: (a) 18 January 2001, corresponding to summer season (from Carvalho [14]) and (b) 29 October 2001, corresponding to spring season (from Carvalho [14]).
coast (Figure 3(b)). Nevertheless, it should be remarked that the constant eddy viscosity used in the experiments could be another additional cause. This point demands a deeper investigation.

In Figure 5(a), the velocity flow solution shows a current in the offshore side (linked to the wind patch), which has contribution from the coastal current from the east. The flow disturbance near the Ilha Grande Bay could be observed in the west side of the region.

Figure 5(b) exhibits the offshore current flow, formed near the coast, due to the confluence of a coastal current from the east and a coastal countercurrent from the west.

The results depicted in the Figure 5(c) differ from those presented in Figure 5(b) mainly because the countercurrent contribution from the west is significantly wider, influencing the SST pattern in such a way that an elongated cold-water band (filament) in offshore direction is formed (see Figure 4(c)).

In Figure 5(d), the velocity field shows a flow spreading in offshore direction. This behaviour is particularly notable in front of Cabo Frio cape. In this particular experiment, the strong wind forcing applied along the west sector of the zonal coastline, leads to calculated velocities which are stronger there.

A common feature in all the velocity field solutions is the occurrence of a flow from the Ilha Grande bay drifted by the wind.

4. SUMMARY AND CONCLUSIONS

In the present paper, the dynamic response of the coastal waters along the Rio de Janeiro coast due to typical northeast wind field forcing patterns was studied. The main hydro-thermodynamical features of cold plumes, coastal jets and observed events of coastal upwelling phenomena were qualitatively described by the proposed two-dimensional gravity reduced model, for which a stabilized space–time Petrov–Galerkin method was employed.

The four performed numerical experiments with distinct idealized northeast wind field patterns showed a remarkable solution dependence on the horizontal structure of the wind in relation to the coastline configuration.

When a wind patch is applied in the east offshore side, the model develops a current offshore advecting the coastal upwelling plume in the same direction. When the centre of the wind patch is applied on the coast (neighbourhood of Cabo Frio cape), the solution shows the generation of a cold plume covering the coastal marine band in the west direction and a current going southwestward in a limited extent. But, when the centre of the wind patch is located offshore, the solution shows an upwelling centre at the coast, and the offshore current generated by the wind leads to a cold-water band extended in southwest direction. When the forcing wind pattern is characterized by an internal low wind patch (higher velocities offshore but smaller velocities in the centre) the solution shows a weak but perceptible advection of coastal water in offshore direction; a feature that was not observed in the other cases.

The warmer temperatures which occur in the bays influence the coastal SST field. All numerical experiments performed confirmed this behaviour. In particular, the warm waters of the Ilha Grande Bay interrupt the colder pattern in the west side of the region.

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